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Mathematica Slovaca, Vol. 49 (1999), No. 1, 117--127

Persistent URL: `http://dml.cz/dmlcz/136744`

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Mathematica Slovaca

PROFESSOR IVAN KOREC DIED

Ivan Korec passed away on August 4, 1998, as a result of a heart attack while on holiday in Duchoňka. We shall sorely miss him; he was a dear friend.

Ivan Korec was born on September 1, 1943 into a family of an official at Chynorany, in the central part of Slovakia. He was educated at a grammar school in Partizánske and finished in 1960. He continued at the Faculty of Natural Sciences of the Comenius University in Bratislava and graduated in 1965.

Ivan Korec spent the next twenty two years as a teacher at the Department of Algebra and Number Theory of the Comenius University. He taught courses in algebra, mathematical logic, set theory, automata theory and theory of recursive functions and computability. In 1967 he received the RNDr. degree, in 1972 he defended his Ph.D. thesis on partial recursive functions, and in 1988 he defended his DrSc. thesis which concerned generalized Pascal triangles. In 1981 he was appointed Associate Professor and since 1993 he became Full Professor at the Comenius University. In 1987, he took a full time position at the Mathematical Institute of the Slovak Academy of Sciences where he worked until his death.

Ivan Korec is one of the best known of Slovak mathematicians. His decision to make his career in mathematics was reinforced by the inspirational Slovak mathematician M. Kolibiar. He obtained the first prize in the Competition of Young Mathematicians of the Association of Slovak Mathematicians and Physicists in 1970. He also received several further awards for scientific and pedagogical work.

Ivan Korec published more than 70 papers and wrote 9 textbooks and chapters of books. His scientific work can be divided into four main parts: elementary number theory, general algebra, logic, and theoretical computer science. He also achieved some results in the theory of real functions. A common feature of his papers is emphasis on computational aspects. The last 15 years of his scientific work were featured by the theory of generalized Pascal triangles. We give here a selective description of his work.

Papers of Ivan Korec concern the following areas of elementary number theory: disjoint covering systems, the $3n+1$ problem, perfect rational cuboids and some other diophantine equations, palindromic squares and Pascal triangles. He always preferred concrete problems.

A disjoint covering system is a partition of the set of all integers into finitely many congruence classes. It can be described as

$$a_1(\mathrm{mod}\, n_1), a_2(\mathrm{mod}\, n_2), \dots, a_k(\mathrm{mod}\, n_k).$$

J. Mycielski and W. Sierpiński [MS] conjectured and Š. Znám [Z] proved that, for an arbitrary modulus n_i, $k \geq 1 + f(n_i)$, where $f(m) = \sum_{i=1}^{r} a_i(p_i - 1)$ if $m = \prod_{i=1}^{r} p_i^{a_i}$ is the standard prime decomposition of m. In the most important paper [10] (quoted in [G], [PO], [BFF], [ZN], [S], etc.) Ivan Korec improved this result to the form

$$k \geq 1 + f(n),$$

where n is the least common multiple of the moduli $n_1, n_2, \dots, n_k$. In [40] he gave a further improvement for the so-called non-natural disjoint covering system (cf. [PO]) in the form $k \geq p_3 + f(n)$, where p_3 is the third smallest prime divisor of n.

The $3n+1$ problem is the following: Let $f(n) = n/2$, if n is even, and $f(n) = 3n+1$, if n is odd. Iteration of f produces sequences of natural numbers. The $3n+1$ conjecture asserts that any such sequence eventually runs into the limiting cycle $(4,2,1)$. This hypothesis is called also the Syracuse problem, Collatz-Kakutani problem, etc. Call the least positive k for which the iteration $f^{(k)}(n) < n$ the stopping time of n, and say that n has infinite stopping time if no such k occurs. The $3n+1$ conjecture can be restated that every integer $n \geq 2$ has a finite stopping time (cf. [LA; p. 5]). R. Terras [T] proved that almost all integers have a finite stopping time. The best result (up to now, cf. [WI; p. 9]) in this direction is the following theorem of Korec [55]: For any $c > \log_4 3$, the set of all n for which there exists k satisfying $f^{(k)}(n) < n^c$ has asymptotic density 1. The paper [50] relates the $3n+1$ problem to certain structural properties of certain finite algebras associated with generalized Pascal triangles (cf. [WI; p. 15]). In the joint paper [44] with Š. Znám, the authors constructed a set of positive integers having the same cycles (without finite part) as the set of all positive integers (cf. [WI; pp. 14–15]). Recent citations of these papers are in the books [DT], [G] and the expository paper [WI].

A perfect rational cuboid is a rectangular parallelepiped of which the lengths of the edges, the face diagonals and the body diagonal are integers. A well known conjecture is that there is no such rational cuboid. Ivan Korec [29], [34] first showed that the least edge of a perfect rational cuboid must exceed 10^6. Extensive searches (cf. [G; pp. 173–181]) have shown that a perfect rational cuboid must have all its edges greater than 333750000. In [49] Ivan Korec has proved the following theorem: Let z be the body diagonal of a perfect rational cuboid, x be its maximal edge and let q be a prime divisor of z, say $z = nq$. Then $n > 11.10^6$, $z > 8.10^9$ and $x > 4.10^9$.

D. Singmaster [SI] stated that for any positive integer d almost all binomial coefficients on the classical Pascal triangle are divisible by d. Ivan Korec in [37] gave the following extension: For arbitrary integers $m > 0$, $n > 0$, $a \geq 0$, and $b \geq 0$ the asymptotic density of those u for which d divides

$$\binom{a+mu+b+nu}{a+mu}$$

is equal to 1. His student F. Marko [M] extend this result to lattice points in the first quadrant contained in a band parallel to a line with an irrational angular coefficient.

In connection with the Banach fixed-point theorem, V. Pták [PT] suggested the study of so-called *small functions*. A function $w\colon (0,T) \to (0,T)$ is said to be small on $(0,T)$ if $x + w(x) + w\big(w(x)\big) + \cdots < \infty$ for all $x \in (0,T)$. A characterization of such functions was given by I. Korec [17], which can be found in the book [PP].

In general algebra we mention two topics:

Let $\mathcal{A}$ be a class of algebras of the same signature, and let $A, B \in \mathcal{A}$. Suppose that for every natural number n, n pairwise disjoint copies of A are subalgebras of B. It is possible that B contains countable many pairwise disjoint copies of A as subalgebras? The answer is *no*, if $\mathcal{A}$ is the class of all posets, graphs, semilattices, lattices, groupoids or algebras with two unary operations, respectively; see [20].

It is now a classical result of A. F. Pixley, that a variety $\mathcal{V}$ of algebras is arithmetical, which means that every algebra A in $\mathcal{V}$ is congruence-distributive and congruence-permutable, if and only if there is a ternary polynomial $p(x,y,z)$ such that

$$p(x,y,x) = p(x,y,y) = p(y,y,x) = x$$

holds identically in every A in $\mathcal{V}$. One can ask if this result is also true for a single algebra A, i.e. without the assumption that A is a member of a variety $\mathcal{V}$. Again Pixley showed in 1972 that this is possible, whenever the algebra A is finite. In 1973, at the Colloquium in Universal Algebra in Oberwolfach, Pixley posed the question of whether the finiteness of A can be omitted. Professor Kolibiar informed Korec about this question. In a series of papers, he settled this problem. First Korec proved in [18] that it is also possible for A to be countable. For uncountable cardinalities, in general, there is no appropriate ternary function $p(x,y,z)$, see [21], [22]. Some modifications or generalizations of this problem can be found in [24], [26] and [28].

From among the extensive variety of Korec's mathematical interests one of the central topics can be broadly demarcated as mathematical logic. Let us mention just three selected highlights of his work in this field — one from the early period, the next two belonging to his recent activities.

In Korec's joint paper with M. G. Peretiatkin and W. Rautenberg [11] the notion of valency (degree of a vertex) is generalized from graphs to arbitrary first-order structures. It is shown that any relation definable in a structure of finite valency is *almost symmetric* in a certain precise sense. This implies the nondefinability of any kind of linear order in a structure of finite valency. It follows that neither addition nor multiplication on the set $\mathbb{N}$ of all natural numbers is definable from any system of relations of finite valency on $\mathbb{N}$.

Later on, in Korec's joint work with Rautenberg [13] the techniques of the previous paper were considerably refined and developed to the extent that it become possible to show the stability in some infinite power for some first-order theories, including for example trees and n-separated graphs. The notion of stability in some infinite power for a first-order theory T is too sophisticated to be explained here, however, it turns out to be equivalent to the nonexistence of a definable linear order on X^n for each $n \geq 1$ and every infinite subset X of any model of T. This also partly reveals one of the essential ideas behind the results mentioned.

These two papers found a deserved response — they are mentioned in works of several prominent mathematicians, such as L. Babai and G. Turán [BT], K. J. Compton and C. W. Henson [CH] (cf. also [BSTW], [HMS], [PZ], [SE]).

Korec's work is difficult to put into boxes. This applies particularly to his pioneering research in generalized Pascal triangles (GPT) which model both the operation structure of regular systolic trellis automata, as well as computation on one-dimensional cellular automata. Given an algebra $\mathbf{A} = (A, *, l, r, c)$ with a binary operation $*$, two unary operations l (for left), r (for right) and a constant c, the GPT associated with $\mathbf{A}$ is the function $\Delta = \mathrm{GPT}(\mathbf{A})\colon \mathbb{N}^2 \to A$ defined in a way that mimics the formation rule of the classical Pascal triangle for binomial coefficients $B(x,y) = \binom{x+y}{x}$, that is,

$$\begin{aligned}
\Delta(0,0) &= c\,,\\
\Delta(x+1,0) &= l(\Delta(x,0)) = l^{x+1}(c)\,,\\
\Delta(0,y+1) &= r(\Delta(0,y)) = r^{y+1}(c)\,,\\
\Delta(x+1,y+1) &= \Delta(x,y+1) * \Delta(x+1,y)
\end{aligned}$$

for all $x, y \in \mathbb{N}$. Then clearly $B = \mathrm{GPT}(\mathbf{N})$ for $\mathbf{N} = (\mathbb{N}, +, l, r, 1)$ with both l and r denoting the function $\mathbb{N} \to \mathbb{N}$ identically equal to 1.

The concept of a GPT, which Korec introduced and elaborated into an elegant mathematical theory, has proved to be fruitful in various branches of mathematics and computer science. We will mention just two examples from the extensive amount of his work in this field, linking GPT's with logic, more specifically, with Peano arithmetic.

A beautiful and slightly weird result from [51] states that addition and multiplication on $\mathbb{N}$ are definable from the usual linear order and almost every relation on $\mathbb{N}$ (in a precise measure-theoretic sense, for a fairly natural probability measure on $\mathcal{P}(\mathbb{N})$).

Korec wrote a series of papers dealing with decidability and definability of arithmetical relations and operations in algebras of the form $(\mathbb{N}, B_n)$ and related structures, where $B_n(x,y) = B(x,y) \bmod n$. His research in this field can be regarded as a precise examination of the borders between decidability and undecidability in arithmetic.

In [54] he showed that, for n divisible by two distinct primes, addition and multiplication are definable in $(\mathbb{N}, B_n)$, hence the elementary theory of this algebra is undecidable. On the other hand, the theory of $(\mathbb{N}, B_2)$ is decidable.

In [57] various definability results focusing mainly around $(\mathbb{N}, B_2)$ were given. In particular, addition and multiplication are definable in $(\mathbb{N}, B_2, \mathrm{Sq})$, where Sq is the set of all squares from $\mathbb{N}$, hence the theory of $(\mathbb{N}, B_2, \mathrm{Sq})$ is undecidable.

In [59] it is shown that the elementary theory of $(\mathbb{N}, B_p, +)$ is decidable for any prime p. On the other hand, $(\mathbb{N}, B_p, \cdot)$ is undecidable.

The final result in this direction is due to A. Bès [B]. Making use of Korec's work he has shown that $(\mathbb{N}, B_q, +)$ has a decidable first-order theory for any prime power q.

Most of the results of Korec in the area of theoretical computer science are related to computability and complexity and very often possess number-theoretic aspects. He regularly attended the theoretical computer science seminar of Jiří Bečvář in Prague and was one of the very first (and very active) members of the Bratislava seminar in theoretical computer science of Jozef Gruska.

His first paper [3] in theoretical computer science introduced a novel type of time complexity of (partial) recursive functions based on the Minsky machines. The class of real-time computable functions introduced there was further studied in his papers [14] and [35].

Korec was always intrigued by problems of "small" machines. For example, in one of his earlier papers [16] he showed an exact boundary for decidability/undecidability of the equivalence problem for Minsky machines with components consisting of at most seven/eight states. In the last years of his life he renewed his interest in problems of this type ([62], [67], [69]).

The major part of his work in theoretical computer science is related to generalized Pascal triangles. Korec introduced this notion in mid eighties and it became the major area of his research until his untimely death. The original motivation for the notion of generalized Pascal triangles was the problem of defining a placement of processors in trellis automata (one of the models of parallel computation introduced in the eighties) which he studied in [33]. His first and most frequently cited paper [38] introduced the notion and gave results on the decidability of the basic problems. Korec remained fascinated by the mathematical difficulty and beauty of problems related to generalized Pascal triangles for the rest of his life.

Korec was a frequent visitor to many parts of Europe, especially as a lecturer at conferences. Here are some selected addresses: Logic Colloquium'94, Clermond-Ferrand, July 1994; Fifth Conference on Discrete Mathematics and Applications, Blagoevgrad, September 1994; Cellural Automata, Wadern, March 1995; Universal Machines and Computations'95, Paris, March 1995; Invited Address, Department Informatique, Universite d'Avergne, Clermont 1, June 1995; Cellular Automata Workshop, Schloss Rauischholz, March 1996; Analytic and Elementary Number Theory Conference, Vienna, July 1996; Number Theory Conference, Eger, August 1996; Invited Address, Instytut Matematyki PAN, Warszawa, June 1997; Invited Address, ÚI AV ČR, Prague, December 1997; International Colloquium Universal Machines and Computations, Metz, March 1998. He was also a regular lecturer at the Czech and Slovak Number Theory Conferences; Czech & Polish Conference on Number Theory, Cieszyn, June 1998.

Korec was currently on the editorial boards of *Mathematica Slovaca* and *Tatra Mountains Mathematical Publications*.

A great part of Korec's professional career was concerned with his pedagogical activities. He was an excellent teacher because of his extreme clarity of mind. During more than thirty years he gave courses in almost all areas of algebra and theoretical computer science, and he wrote textbooks and lecture notes for students. Further, he was involved in the dissertation research of many students and served as the principal advisor for Karol Habart. For a long time Korec was a very active person in the organization of Mathematical Olympiad in Slovakia.

During his scientific and pedagogical work Korec received many appreciations and awards, most recently in 1993 the silver honourable plaquette of Jur Hronec of the Slovak Academy of Sciences for his achievements in mathematical sciences.

We shall all miss him as a talented mathematician and a good colleague. And we shall also remember him with affection.

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