Indiscernibles and Satisfaction Classes in Arithmetic

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Abstract. This talk focuses on the theory PAI (Peano Arithmetic with Indiscernibles), a theory extending Peano Arithmetic that is formulated in the language obtained by adding a unary predicate $I(x)$ to the language of PA. Models of PAI are of the form $(\mathcal{M}, I)$, where $\mathcal{M}$ is a model of PA, I is an unbounded set of order indiscernibles over $\mathcal{M}$, and $(\mathcal{M}, I)$ satisfies the extended induction scheme for formulae mentioning I. As indicated by the following results, there is a close connection between models of PAI and satisfaction classes.

Theorem A. *Let $\mathcal{M}$ be a nonstandard model of* PA *of any cardinality. $\mathcal{M}$ has an expansion to a model of* PAI *iff $\mathcal{M}$ has an inductive partial satisfaction class.*

Theorem A yields the following corollary, which provides a new characterization of countable recursively saturated models of PA:

Corollary. *A countable model $\mathcal{M}$ of* PA *is recursively saturated iff $\mathcal{M}$ has an expansion to a model of* PAI.

Theorem B. *There is a sentence α in the language obtained by adding a unary predicate $I(x)$ to the language of arithmetic such that given any nonstandard model $\mathcal{M}$ of* PA *of any cardinality, $\mathcal{M}$ has an expansion to a model of $\mathrm{PAI} + \alpha$ iff $\mathcal{M}$ has a inductive full satisfaction class.*

The talk is based on a preprint available on arXiv: `https://arxiv.org/abs/2212.08411`